Free Response Questions

Q: 1 Mumbai has a public transport system consisting of local trains which connect [1] numerous stations across Mumbai. Among them are Andheri and Dadar.

Vani takes the local train scheduled at 8:55 AM from Andheri station to Dadar station every morning.

- The probability that the train is late is $\frac{3}{4}$.
- The probability that Vani gets a seat in the train is $\frac{1}{15}$.

What is the probability that the train is on-time, and Vani gets a seat in it? Show your work.

Q: 2 A and B are events in the sample space S such that P(A) > 0 and P(B) > 0. [1]

Check whether the following statement is true or false. If true, justify your conclusion; if false, state the right expression.

"If A and B are mutually exclusive, the probability of at least one of them occurring is: P(A).P(B)"

Q: 3 A company conducts a mandatory health check up for all newly hired employees, to [3] — check for infections that could affect other office-going employees. A blood infection affects roughly 5% of the population. The probability of a false positive on the test for this infection is 4%, while the probability of a false negative on the test is 3%.

If a person tests positive for the infection, what is the probability that they are actually infected? Show your work.

(Note: A false positive on a test refers to a case when a person is not infected, but tests positive for the infection. A false negative on a test refers to a case when a person is infected, but tests negative for the infection.)

Q: 4 A survey says that approximately 30% of all products ordered online across various [5] — e-commerce websites are returned. Two products that are ordered online are selected at random.

If X represents the number of products that are returned,

- i) find the probability distribution of X.
- ii) find the mean or expectation of X.

Show your work.

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Q: 5 Akshay is solving an exam paper with 50 multiple choice questions, each for 1 mark. In ^[5] the paper, each multiple choice question has 4 options and exactly one correct answer.

Akshay solves only 30 questions, and randomly guesses the answers for the rest.

If exactly 40% of the questions Akshay solved are solved correctly, then what is the probability that Akshay scores at least 30 marks on the exam? Show your work.

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Q.No	What to look for	Marks
1	Finds the probability that the train is on-time as $1 - \frac{3}{4} = \frac{1}{4}$.	0.5
	Finds the probability that the train is on-time, and Vani gets a seat in it as $\frac{1}{4} \times \frac{1}{15} = \frac{1}{60}$.	0.5
2	Writes false.	0.5
	Writes the correct expression. For example, since A and B are mutually exclusive, A \cap B = Φ , which means that P(A \cap B) = 0. Hence, the probability that at least one of them occurs is P(A \cup B) = P(A) + P(B).	0.5
	(Award full marks if just P(A \cup B) = P(A) + P(B) is written without any explanation.)	
3	Considers:	0.5
	X = Event that employee is affected by the infection. Y = Event that employee tests positive for the infection.	
	Writes that:	
	P(Y X') = 0.04 P(Y' X) = 0.03 P(X) = 0.05	
	Writes that:	0.5
	P(Y X) = 1 - P(Y' X) = 0.97	
	Uses the law of total probability to find P(Y) as follows:	1
	P(Y) = P(Y X)P(X) + P(Y X')P(X') => P(Y) = (0.97)(0.05) + (0.04)(0.95) => P(Y) = 0.0485 + 0.038 = 0.0865	

Q.No	What to look for	Marks
	Uses Bayes' Theorem to find P(X Y) as follows:	1
	$P(X Y) = \frac{P(Y X)P(X)}{P(Y)}$	
	$\Rightarrow P(X Y) = \frac{0.97 \times 0.05}{0.0865} = \frac{0.0485}{0.0865}$	
	$\Rightarrow P(X Y) = \frac{97}{173}$	
4	i) Writes that the possible values of X are 0, 1 and 2.	0.5
	Takes R to be the event that the product is returned and N to be the event that the product is not returned.	1
	Uses the given information and finds the probability that the product is returned as $P(R) = 0.3$ and the probability that the product is not returned as $P(N) = 1 - 0.3 = 0.7$.	
	Finds the probability of each possible values of the random variable X as:	2
	P(X = 0) = P(NN) = (0.7)(0.7) = 0.49	
	P(X = 1) = P(RN, NR) = (0.3)(0.7) + (0.7)(0.3) = 0.21 + 0.21 = 0.42	
	P(X = 2) = P(RR) = (0.3)(0.3) = 0.09	
	Finds the probability distribution of X as:	0.5
	X or x _i 0 1 2	
	P or p _i 0.49 0.42 0.09	
	ii) Finds the mean or expectation of X as:	1
	$E(X) = \sum x_i p_i = (0 \times 0.49) + (1 \times 0.42) + (2 \times 0.09) = 0.6$	

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Q.No	What to look for	Marks
5	Finds the number of questions Akshay solved correctly as 40% of 30 = 12.	1
	Finds the number of questions Akshay randomly guessed the answer for as 20.	1.5
	\therefore The number of trials, $n = 20$	
	Assumes <i>p</i> as the probability that the answer is correct, and <i>q</i> as the probability that the answer is incorrect.	
	Finds $p = \frac{1}{4}$ and $q = \frac{3}{4}$.	
	Finds that the required probability, P(at least 30 marks) = P(at least 18 correct guesses) = P(exactly 18 correct guesses) + P(exactly 19 correct guesses) + P(exactly 20 correct guesses).	0.5
	Evaluates this using the Binomial Distribution as follows:	1.5
	P(exactly 18 correct guesses) = ${}^{20}C_{18} \left(\frac{1}{4}\right)^{18} \left(\frac{3}{4}\right)^2$ = $\frac{20!}{18! \times 2!} \times \frac{9}{4^{20}} = \frac{1710}{4^{20}}$	
	P(exactly 19 correct guesses) = ${}^{20}C_{19} \left(\frac{1}{4}\right)^{19} \left(\frac{3}{4}\right)^{1}$ = $\frac{20!}{19! \times 1!} \times \frac{3}{4^{20}} = \frac{60}{4^{20}}$	
	P(exactly 20 correct guesses) = ${}^{20}C_{20}\left(\frac{1}{4}\right)^{20}\left(\frac{3}{4}\right)^{0} = \frac{1}{4^{20}}$	
	Finds the required probability as:	0.5
	$\frac{1710}{4^{20}} + \frac{60}{4^{20}} + \frac{1}{4^{20}} = \frac{1771}{4^{20}}$	

